Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER– III (NEW) EXAMINATION – SUMMER 2022

Subject Code:2130002 **Date:08-07-2022**

Subject Name:Advance Engineering Mathematics

Time:02:30 PM TO 05:30 PM **Total Marks:70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

			MARKS
Q.1	**(a)**	Solve the differential equation $ye^x\,dx+\left(2y+e^x\right)dy=0$.	**03**
	(b)	Find the Laplace Transform of $(t+1)^2 e^t$	**04**
	(c)	Find the Fourier series expansion of the periodic function $f(x)=x-x^2$ in the interval $-\pi\le x\le\pi$.	**07**
Q.2	**(a)**	Find $L^{-1}\left\{\dfrac{3s+4}{s^2+9}\right\}$	**03**
	(b)	Solve $y''-2y'+y=10e^x$	**04**
	(c)	Find the Fourier series of the periodic function with a period 2 of $f(x)=\begin{cases}\pi\ , & 0\le x\le 1\\ \pi(2-x)\ , & 1\le x\le 2\end{cases}$	**07**
		OR	
	(c)	Find the power series solution of the equation $(x^2+1)y''+xy'-xy=0$ about $x=0$.	**07**
Q.3	**(a)**	Define Beta function, Gamma function and write the relation between Beta and Gamma function.	**03**
	(b)	Use convolution theorem to find the inverse Laplace Transform of $\dfrac{1}{(s+1)(s^2+1)}$.	**04**
	(c)	Solve the differential equation using method of variation of parameters: $y''+9y=\tan 3x$.	**07**
		OR	
Q.3	**(a)**	Define (1) Rectangle function; (2) Saw tooth wave function .	**03**
	(b)	Find the half range sine series of $f(x)=x^2$ in the interval $(0,\pi)$.	**04**
	(c)	Solve the initial value problem using Laplace Transform $y''+y'=t^2+2t$, $y(0)=4,\ y'(0)=-2$	**07**

Q.4 **(a)** Find the Laplace Transform of $\int_0^t e^{-2t}\, t^3\, dt$. **03**

(b) Solve the differential equation $\frac{dy}{dx}+\frac{2y}{x}=y^2x^2$. **04**

(c) Find the Fourier Integral representation of the function **07**

$$f(x)=\begin{cases}1-x^2\ , & |x|\le 1\\ 0\ , & |x|>1\end{cases}$$

OR

Q.4 **(a)** Find the inverse Laplace Transform of $\frac{e^{-\pi s}}{s^2-2s+2}$. **03**

(b) Solve $(mz-ny)p+(nx-lz)q=ly-mx$. **04**

(c) Solve $x^2y''+5xy'+3y=\frac{\log x}{x^2}$ **07**

Q.5 **(a)** Form a differential equation for the equation $z=(x-2)^2+(y-3)^2$. **03**

(b) Find the Laplace Transform of $f(t)=\begin{cases}t^2\ , & 0<t<1\\ 4t\ , & t>1\end{cases}$ **04**

(c) Solve the equation $u_x=2u_t+u$ given $u(x,0)=4e^{-4x}$, by the method of separation of variables. **07**

OR

Q.5 **(a)** Solve $p^2+q^2=x+y$ **03**

(b) Solve $\frac{\partial^2 z}{\partial x^2}+z=0$, given that when $x=0$, $z=e^y$ and $\frac{\partial z}{\partial x}=1$. **04**

(c) Solve $(D^2+DD'-6D'^2)z=\sin(2x+y)$. **07**

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